Title: Calculating time dilation within general and special relativity
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Non-kinematical clock-slowing by way of Newton's law of gravity

Computing kinematical and non-kinematical time dilation.

Inertial frame reference points

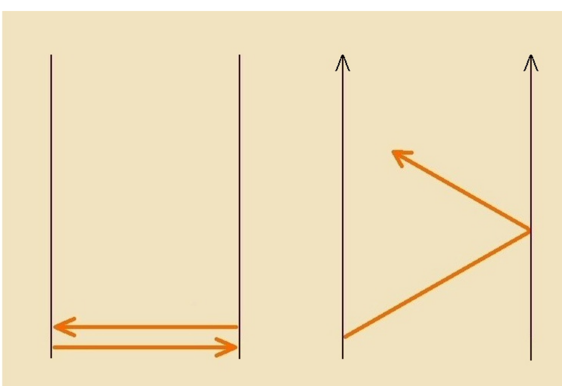
Hafele-Keating circumnavigating jets study,
Around-the-World Atomic Clocks: Predicted Relativistic Time Gains

Scott Kelly's International Space Station (ISS) mission, and his lesser aging than that of his twin brother Mark Kelly who remained on Earth

Hierarchy of speed and hierarchy of clock rates

Absolute frame of reference

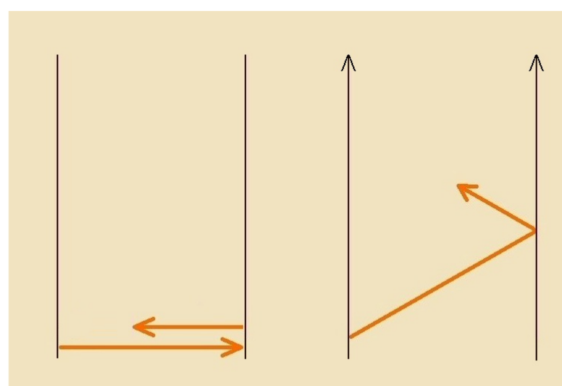
Einstein's 1920 Leyden lecture



at rest and no gravity

uniform linear motion and no gravity

kinematical only



at rest and gravity

uniform linear motion and gravity

kinematical plus non-kinematical

Above: Newton's law of gravity dictates the slowing of light (which dictates clock-rates).

Combined with the equivalence principle and the simple kinematical clock-slowing of uniform motion, the resulting set of equations precisely match Einstein's treatment for gravitational clock-slowing associated with the gravitational field of a non-rotating massive body.

Calculating time dilation within general and special relativity

Understanding and calculating non-kinematical and kinematical time dilation

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Easily compute a net or total time-keeping difference by combining GR and SR equations
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The non-kinematical effect is referred to as the GR (general relativity) effect.

Calculating the GR effect is a two-step process. The first step (eq 1 below) yields the escape velocity for an object at a particular distance from the center of a given gravitational field. The second step (eq 2) utilizes that escape velocity to compute the GR portion of the time-keeping dilation.

(We refer to the combined effect of GR and SR as the total time difference when they have an additive effect. We refer to the combined effect as the net effect when the two effects cancel to some extent.)

Note that even though we call the non-kinematical effect the general relativity effect, the equations below are actually derived solely from Newton's law of gravity combined with the very simple equivalence principle and the very simple kinematics of uniform motion of special relativity. It is nothing more than basic freshman physics, in keeping with the methodology described above.

The identical non-kinematical equation is generated by the general relativity Schwarzschild metric, which is the solution to Einstein's field equations for a non-rotating, spherically symmetric mass.

It is popularly considered a coincidence that the two approaches generate the identical equation, but not by me. See: spacetime curvature.

General Relativity time-keeping dilation:

```
-------------------------------------------------------------
G is the gravitational constant (empirically established).
M is the mass of the body generating the gravitational field.
r is the distance of the object from the center of gravity.
ev is the escape velocity of the object, based on r and M.
c is the speed of light.
-------------------------------------------------------------
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$$G = 6.674(10^{-11})(m^3)(kg^{-1})(s^{-2})$$

$$ev = (2GM/r)^{1/2} \qquad \text{[eq 1]}$$

$$GR\ dilation = (1 - (ev/c)^2)^{1/2} \qquad \text{[eq 2]}$$

Inserting G, M (mass of the earth) and r into eq 1 we have:

$$ev = \frac{(2\ (6.674)\ m^3\ (5.972)\ 10^{24}\ kg)^{1/2}}{(10^{11}\ kg\ s^2\ r\ m)^{1/2}}$$

After multiplying, canceling the exponents
and taking the square roots, we have simply:

$$ev = \frac{28,233,713}{r^{1/2}}\ m/s$$

where r is in meters.

If all of your computations involve the earth's gravitational field (clocks at sea-level and on airplanes, tethered balloons, mountain summits, orbiting spacecraft and satellites), you can just stick with 28,233,713 in the numerator.

(If you want to express r in km, then the numerator is 892,828 which makes for fewer digits twice over. And ev is still expressed in m/s.)

Then all you have to do is insert a value for r (distance from the center of the earth for any of the above-mentioned objects) to get the escape velocity (ev) for that radius.

Note that you must calculate the GR effect both for a clock on the surface of the earth (which is typically where a reference clock is) and for the clock on the (say spacecraft). You are comparing each of those clock rates to an imaginary clock at infinite distance which is not under the influence of gravity. (In reality, that is a clock in a higher dimension, free of gravity.)

You then take the difference between the GR effect on the spacecraft and the GR effect on the clock on the surface of the earth. If you calculate correctly, you should obtain a value of 0.021956 second (GR effect) lesser time for the Earth-surface clock over the course of a year as compared to the infinite distance clock when using Earth radius = 6374 km and Earth mass of 5.972 10^24 kg.

As the above-mentioned entities are on or near the surface of the earth, one doesn't need to factor in the gravitational effect of the moon.

When there is more than one massive body involved, the GR effect generated by each gravitational field is fully additive, even though the gravitational attractions of opposing bodies have a canceling effect on weight.

One needs to apply a bit of calculus or do some averaging over the course of a trip if an entity's experience of field strength varies. That of course would apply to trips to the moon.

Similarly, one needs a bit of calculus or an approximation technique if one wants to increase computational accuracy by considering the kinematical effect variation during an acceleration phase.

Next, we combine the GR dilation:

```
GR:   dil = (1 - (ev/c)^2)^1/2    [eq 2]
```

with the kinematical dilation of Special Relativity to obtain the net (or the total if they are additive) time dilation.

I've always written the kinematical time-keeping dilation equation as:

```
SR:   dil = (1 - (v/c)^2)^1/2
```

where dil is the multiplier for any given time interval as recorded by a reference clock that is in an inertial frame (practically speaking) and is therefore conveniently considered to be at rest. See the example below and the discussion which follows for more considerations about inertial reference points.

The text books write the equation as:

```
              d
d' = ---------------------
     (1 - (v^2 / c^2))^1/2
```

Same is true for the way I wrote eq 2 for the GR dilation.

Example:

Scott Kelly spent 340 days onboard the International Space Station.

We will assume that his brother Mark remained at about 45 degrees latitude.

Reference point for kinematical considerations is the center of the earth.

Reference point for non-kinematical considerations is a point at infinity.

```
Average radius of Earth               6,371,000 m
Average altitude of ISS                 405,000 m
Average radius for ISS                6,776,000 m
Average circumference for ISS        42,574,864 m
Average orbital period of ISS             5,478 s
==> average speed of ISS                  7,772 m/s
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Speed of ground at 45 degrees                328 m/s
Speed of light                       299,792,458 m/s
Escape velocity at 6,371,000 m            11,186 m/s
Escape velocity at 6,776,000 m            10,846 m/s
Mission duration                      29,376,000 s

GR slowing for Mark vs ref         0.0204489 s
GR slowing for Scott vs ref        0.0192247 s

GR net: Mark ages 0.0012242 second less than Scott

SR slowing for Mark vs ref        0.00001756 s
SR slowing for Scott vs ref       0.00987156 s

SR net: Scott ages 0.009854 second less than Mark

Combined GR/SR net:

Scott ages 0.0086 second less than Mark

0.0086 second = 8.6 milliseconds
```

Discussion

Inertial frame reference points

We don't always consider a clock on the earth's surface to be at rest. When considering airplanes rather than fast-moving spacecraft, we can make meaningful percentage-wise comparisons only by considering both the airplane's speed and the earth's surface rotational speed relative to a "fixed" reference point, such as the center of the earth. We then compare the kinematical time-keeping dilation of the Earth-surface clock to the kinematical time-keeping dilation of the airplane's clock.

To make a more accurate prediction about kinematical time-keeping dilation for spacecraft, we would also need to proceed as described in the previous paragraph. Since spacecraft have a far greater speed than the Earth-surface speed, the accuracy of the prediction is only very slightly improved (percentage-wise); so in that case it doesn't much matter whether we consider the Earth-surface speed relative to a fixed point.

It makes sense to regard the center of the earth as a fixed point since it is not changing frames relative to the orbital path of the entities that revolve around it. To analyze the kinematics of the spiral path that is generated by orbiting the earth as the earth orbits the sun would be daunting and would not change anything.

One can extend that logic all the way to the barycenter of the universe.

Or, see: relativitytrail.com/centered.pdf (barycenter or expansion of space?)

Hafele-Keating circumnavigation study

The first and still most famous example is the Hafele-Keating theoretical and experimental study of 1971 involving two jets traveling in opposite directions as they circumnavigated the globe. The rotational speed of the ground station containing the clock on the surface of the earth relative to the center of the earth and its resultant time-keeping dilation was taken into consideration.

Their paper lacked details on the altitudes and speeds of the jets in the experimental study, but did include a graph showing theoretical circumnavigation at the equator involving jets flying in opposite directions at various altitudes and speeds. I made the GR and SR calculations for those examples and obtained the same time-keeping dilation results as shown on the graph.

Fixed vs non-fixed reference point

You might wonder why one can't just regard one of the two jets to be the fixed point of reference and then calculate the kinematic time dilation between them.

Wouldn't work for two reasons. Whichever jet you choose as the fixed point of reference would have the greater predicted recorded time upon reuniting – an obvious absurdity since of course only one of them can show a greater recorded time than the other upon reuniting.

In fact, you will not obtain the correct absolute value of the time dilation: Both jets are changing frames relative to all elements in the study due to their circular motion, whereas the center of the earth is not changing frames relative to the orbital path of the entities that revolve around it, as we previously indicated.

That fits with both the absolutist and relativist approach to special relativity. As always, the absolutist and the relativist are in agreement when it comes to predicting the time-keeping difference *upon the reuniting of the clocks*, just as they are in agreement about symmetrical assessments across inertial frames and the constancy of the measured speed of light. That is, they agree that there is no privileged frame of reference in which to conduct experiments.

Absolute frame of reference (the universe)

Both the circumnavigating jet study and the experimental evidence obtained from satellites which show kinematical differences in time-keeping plainly reveal that there truly is a hierarchy of clock rates dependent on a hierarchy of speed in the context of the universe.

Form a picture in your mind and try to imagine that the difference in speed between an orbiting spacecraft and a point on the surface of the earth is not actual in the context of the universe, which is the ultimate frame of reference.

That is not merely a concept of convenience. Rather, the totality of the universe – in strict accordance with the reasoning of Mach and others – imparts physical properties (clock-rate-contraction and length-contraction) to everything it contains. As Einstein plainly stated at Leyden in 1920, “Mach's idea finds its full development in the ether of General Relativity” and “Space without ether is unthinkable; for in such space there .. would be .. no possibility of existence for standards of space and time, [specifically] our measuring-rods and clocks, nor therefore any [space or time] intervals in the physical sense.” [1]

Speed is not merely relative. Strict relativists (wrongly) regard speed to be merely relative due to our symmetrical assessments of properties across inertial frames. They are conflating "symmetry of measure" with "no truth of the matter".

Tacit acknowledgement

Notably, in the Hafele-Keating study, the authors are tacitly acknowledging the pertinence of the reference frame of the universe, whether they are at all conscious of it or not.

The atomic clocks did show a lesser time for the jet with the greater speed relative to center of the earth than for the jet with the lesser speed relative to the center of the earth. Hafele and Keating regard both jets as moving faster than the center of the earth. That is, whatever speed the center of the earth has relative to the totality of the cosmos, the jets have an even greater speed. If Hafele and Keating did not make such consideration, they would not have correctly predicted the experimental result. The hierarchy of speed is in plain evidence.

That applies to linear motion with respect to the totality of the cosmos as well:

Though you will come across it many times, it is absurd to claim that it is just as meaningful to consider that the earth, along with the entire universe, has changed inertial frames to facilitate a reunion with an astronaut as it is to consider the astronaut as having made the inertial change.

Does anyone really think that a clock (astronaut) can impart (beyond an infinitesimal) physical properties to the universe, rather than the other way around?

And only one of those entities can age less than the other.

In the experiment with the jets, indeed, the one with the greater speed relative to the universe recorded less time than the other jet.

1. Einstein, A. Ether and the Theory of Relativity. English translation from the German. (1922). Methuen & Co. Ltd, London. https://mathshistory.st-andrews.ac.uk/Extras/Einstein_ether

See: Around-the-World Atomic Clocks: Predicted Relativistic Time Gains
- J. C. Hafele, Richard E. Keating

https://relativitytrail.com/hafele-keating/index.htm

See: Understanding what generates symmetrical measures and the time differential in special relativity
- Roger Luebeck

https://relativitytrail.com/preprint.special_relativity.pdf

See sitemap for my other articles:

Symmetry of measure – Journal article preprint

Fallacy of strict relativism – in relativity

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Einstein at Leyden – Standards for clocks and rods

Spacetime curvature - Why it works in GR

Computing GR and SR time-keeping dilation – with discussion

Spacetime – is a mathematical construct

Twins paradox animation – and simple equation

Citation and annotation for the book *Relativity Trail*